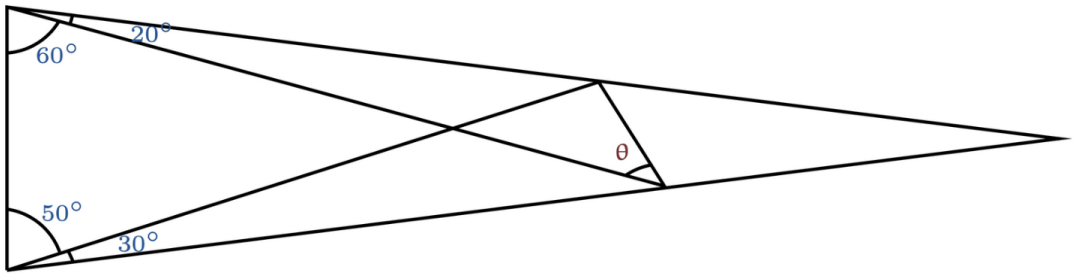
NMSU MATH PROBLEM OF THE WEEK

Solution to Problem 9

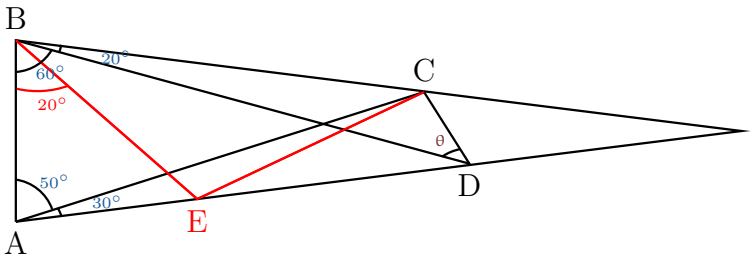
Fall 2025

Problem 9

Find the angle θ in the diagram below. Justify your answer:



Solution. In the given diagram, we make additional constructions and label the vertices as shown below:



Claim. $|AB| = |BE| = |BC| = |CE| = |DE|$.

Proof. Consider the triangle ΔABE. Since sum of the angles of a triangle must equal $180°$, we conclude $\angle AEB = 80°$. Thus, ΔABE is an isosceles triangle with

$$|AB| = |BE|. \tag{1}$$

Now we consider the triangle ΔABC. In this triangle, $\angle ABC = 80°$ and $\angle BAC = 50°$. Therefore,

$$\angle ACB = 180° - (80° + 50°) = 50°$$

and ΔABC, is also an isosceles triangle with

$$|AB| = |BC|. \tag{2}$$

Next, we consider ΔBCE. By construction $\angle$CBE $= 60°$, and from (1) and (2) we know that $|\text{BE}| = |\text{BC}|$. Therefore,

$$\angle\text{BEC} = \angle\text{BCE} = (180 - 60)/2 = 60°.$$

Since, all angles in ΔBCE are equal, it must be an equilateral triangle, and therefore,

$$|\text{BE}| = |\text{BC}| = |\text{CE}|. \tag{3}$$

Finally, we consider ΔBED. Note that $\angle$BED $= 180° - \angle$AEB $= 100°$, and $\angle$DBE $= 40°$ by construction. Therefore,

$$\angle\text{BDE} = 180° - \angle\text{BED} - \angle\text{DBE} = 40°. \tag{4}$$

Thus, ΔBED is also an isosceles triangle with

$$|\text{BE}| = |\text{ED}|. \tag{5}$$

The claim then follows from (1), (2), (3), and (5). □

To find the angle θ, we consider ΔCDE. This triangle is an isosceles triangle as $|\text{CE}| = |\text{DE}|$ from our claim above. Morever, $\angle\text{CED} = 180° - \angle\text{BEC} - \angle\text{AEB} = 40°$. Consequently

$$\angle\text{CDE} = \angle\text{DCE} = (180 - 40)/2 = 70°.$$

Since $\angle\text{CDE} = \theta + \angle\text{BDE}$ and $\angle\text{BDE} = 40°$ by (4), we conclude that $\theta = 30°$. ■